MODEL OF GREEN EXTENSION TRAFFIC SIGNAL CONTROL SYSTEM USING PETRI NET

Tomi Tristono[1], Setiyo Daru Cahyono[2], Sudarno[3], and Pradityo Utomo[4]
[1]University of Merdeka Madiun – Indonesia, Serayu Road 79 – Madiun – East Java – Indonesia
email: tomitristono@gmail.com
[2]University of Merdeka Madiun – Indonesia, Serayu Road 79 – Madiun – East Java – Indonesia
email: cahyono.ds@gmail.com
[3]University of Merdeka Madiun – Indonesia, Serayu Road 79 – Madiun – East Java – Indonesia
email: sudarno@unmer-madiun.ac.id
[4]University of Merdeka Madiun – Indonesia, Serayu Road 79 – Madiun – East Java – Indonesia

Abstract

This paper describes a method for constructing a control system of the traffic signal of an intersection in the urban road networks. The implementation of this traffic signal can optimize the performance of the intersection, reduce travel delays, avoid congestion, and prevent conflict of traffic flow. The traffic signal applies additional green signals as the extensions, i.e., the four-seconds extra. The additional green signals are implemented at most four times. Petri Net (PN) is usedto improve the control system of the traffic signal.This method can present a mathematical concept of a control system in Discrete-Event System (DES). For model verification and its validation,invariants and simulations are requisite. It represents the marking on the places and indicates a sequence of events in the actual state, respectively. In this paper presents the comparison of the travel delays while applying additional green signals once, twice, and three times after a barrier has elapsed. The control system is proportional to the number of constraints that must be satisfied.

Keywords: *control system, additional green signals, Petri net, travel delays.*

1. INTRODUCTION

Discrete Event Systems (DES) is very suitable to model the behavior of traffic lights signals. It can formulate events based on its sequences. One of DES manifestations is expressed by Petri Net (PN) which consists of four elements those are places, transitions, arcs, and tokens [1],[2]. PN is appropriate to model the traffic signal because it offers a representation of conflict situations, share travel schedule allocations, synchronization, non-synchronization, and various variables that have to be prioritized [3]. PN also can construct the controllers for sequencing events with quick response time in real-time control [4].

This paper introduces a method for computing a feedback controller of traffic signal behavior as a Discrete-Event System. For this purpose, PN is used. It is based on the reason that the computational of PN involves less than the multiplication of a single matrix [5]. The computational efficiency makes it easy to apply as a method in the practical approach to creating a controller synthesis of a large and complicated Discrete-Event Systems.

The traffic signals are implemented at an intersection in the urban road network. This traffic signal can adapt to an additional barrier that exists at the intersection, which is a railway crossing in the southern arm. As the aim of this study, we created the traffic signals that must be able to adapt to the constraints of railway doorstop closure to reducethe travel delays.

When the detector takes information that the train will arrive soon, the synchronous control system sends a command to the south arm signal to convert to red immediately. The southern signal is red while the train is passing. On the contrary, the eastern signal is always green. For the safety of travel, the train is always placed as a priority.

Especially while peak hour,the constraint of railway doorstop closure always causescongestion that leads to a substantial degradation [6], causes inefficiency, the inconvenience of travel, and resulting in economic losses [7]. The ordinary of synchronous traffic signals on the railway doorstop has not provided the maximum results [8],[9]. The railway doorstop always takes a long time. The vehicles that have been stopped by trains require the control system of additional green traffic signals to decrease the travel delay. It is very urgent.

Many previous researchers had studied the dynamics of behavioral systems using Petri net. The colored timed Petri nets were used by Dotolia et al. for reviewing an urban traffic network model [10]. Petri Nets was used by Alast to analyze the workflow of management models [11]. The timed colored of Petri nets wereused by Bozek to model, simulate, and to schedule the production systems [12]. Bordon et al. wrote the paper the fuzzy logic and Petri nets as the design to overcome the unknown kinetic data for the quantitative modeling of the biological systems [13]. The study of the modeling and analysis of traffic light control systems had done by Huang, et al. Thetimed colored Petri net was used [14]. The research of Mladenovic et al. about modeling ring-barrier traffic controllers. The colored timed stochastic Petri nets wereused [15].

2. MATERIAL AND METHOD

In this section is written about traffic signal modeling methods. It contains subsection Petri net contexture, an intersection and its traffic flow, verification and validation of the model, the additional green signals time interval, and the travel delay.

2.1 Petri Net Contexture

Petri Net Contexture. As the graphically mathematical language, Petri nets can represent the events flow of modeling dynamics in parallel, asynchronous, simultaneous, distributed, and stochastic [3]. The Petri nets contexture is formalized as the following. The contexture of the classic (timeless) Petri net has four elements, i.e., N = (P, T, A, w). P is a finite set of places which is illustrated using circles/ ovoid. T is a finite set of transitions which is represented using a rectangle/ square. $P \cap T = \emptyset$. A is a setof arcs as a relationship on places to transitions or transitions to places. The model of mathematical expression is $A \subseteq (P \times T) \cup (T \times P)$. The weight function w: $A \rightarrow \{0,1,2,3, ...\}$. A marking of N is a function M: $P \rightarrow \{0,1,2,3, ...\}$, which each place $p \in P$ is assigned by the number of tokens. The initial marking is Mo. It shows the number of tokens that belong to each place at the beginning [2].The Petri net desires the properties to identify the presence or absence of its function in the design system.

2.2 An Intersection and Its Traffic Flow

An Intersection. Fig. 1 is the illustration of a signalized intersection consisting of four arms. Both one-way movement of the traffic on the east arm and the south arm entering the intersection. The traffic flow of vehicles on both the western and northern arms move in one direction also. They keep away from the intersection. A railway track crossing is in the southern arm.

Its Traffic Flow. The flow setting of the vehicle implements two-phase traffic signals. The model is represented in Fig. 2., and the schedule is in Table 1. The railway doorstop model is not shown. While a train is passing, a token exists in each of the eastern green place and southern red place. It is synchronization between traffic signals to the railwaydoorstop. This is based on information from rail detectors.

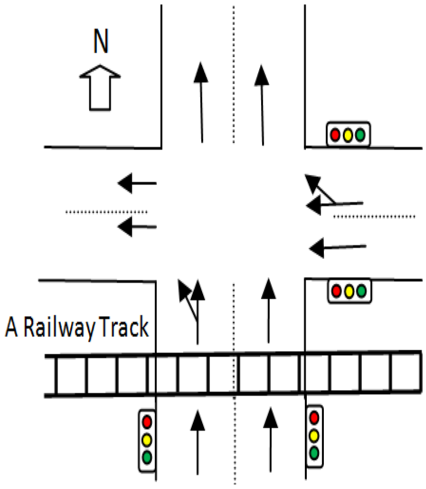

Fig. 1. The figure of a signalized intersection, traffic flow, and a railway crossing in the southern arm. The railway doorstop closure is always longer than the time interval of a traffic signal cycle [16].

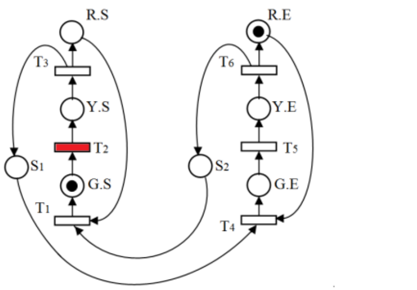

Fig. 2. Petri net model of the two-phases traffic signal.

A token is in each place G.S (Green-South) and R.E (Red-East). The enabled transition T_2 ready for firing to transfer a token from G.S to place Y.S (Yellow-South). While a token exists in a place G.S, the meaning is the green signal on the southern arm is turning on. The places G.E and R.S are Green-East and Red-South, respectively. Place S_1 and S_2 are places intermediaries, which are tasked to create a synchronized signal sequence on the south arm and the east arm.The place Y.E is Yellow-East.

Table 1. Two-phases traffic signals. The south arm applies the minimum green signals time interval.

Phases	Green	Inter Green		Red	Cycle
		Yellow	All red		
South Arm	24	3	3	45	72
East Arm	36	3	3	33	72

Fig. 3. is a simulation of the Petri net model from Fig. 2. This is a regular traffic signal setting using a fixed time strategy. While the train is passing, the G.E interval becomes very long. It depends on the time interval of the railway door stop closure.

2.3 Model Verification and Validation

Model verification and validation.The firing of the enabled transitions produces a sequence of marking of a Petri net. All reachabled marking have several properties that ensure firing a transition will not produce the different outputs [3]. It is expressed as an invariant formula. The marking of the place is "1" or "0" only, it's turning on or off, respectively. The invariantsare derived from the adjacency matrix of the Petri net model structure.

$$M(G.S)+M(Y.S)+M(R.S)=1 \qquad (1)$$

Invariant (1) states that there is only one token in one of three places G.S, Y.S, and R.S. The traffic signalsallow one active signalonly. Whenthe green south signal is on, the other signals have to be off, i.e., red south and yellow south. It means that the marking of place green south is one, the marking of both places red south and yellow south are zero.When the red south or yellow south signal is active,a similar methodis used. The Invariant (2) for east arm traffic has a meaning identicalto Invariant (1).

$$M(G.E)+M(Y.E)+M(R.E)=1 \qquad (2)$$

While there is a token in place R.E, there must be a token in one of the places G.S, Y.S, or R.S. It is indicated ininvariant (3).The south signal may be green, yellow, or redwhile the eastern signal is on. This method is usedto separate the traffic flows coming from the east or south to avoid conflict.

$$M(G.S)+M(Y.S)+M(R.S)=M(R.E)$$
$$while\ M(R.E)=1 \qquad (3)$$

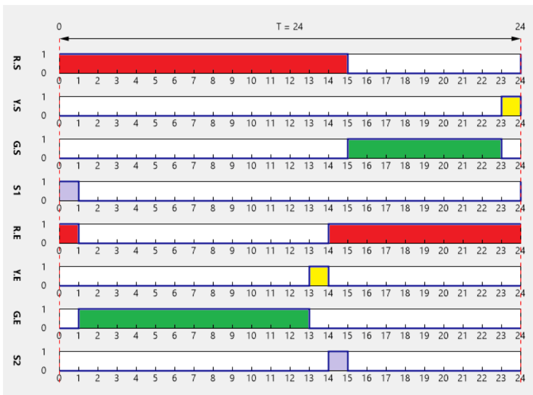

Fig. 3. The illustration of the traffic signal simulation result that implements minimum green signal time interval on the south arm.

The minimum green signal time interval is 24 seconds. Each step represents three seconds. When both green and yellow signals of the southern arm are lit, the red signal of the east arm must be turned on. While both green and yellow signals of the eastern arm are active, the red signal of the southern arm is turning on. When a token is existing in S_1 or S_2, traffic signals on both arms are turning on the all red signal. This is the time interval to empty the intersection to ensure that no conflict of vehicle traffic flows [16],[17]. For the simulation, Petri net Simulator 2.0is used. The invariant(4) is in the same method as the invariant(3).

$$M(G.E)+M(Y.E)+M(R.E)=M(R.S) \quad while \;\; M(R.S)=1 \tag{4}$$

Invariant (5) is equipped withintermediary places, i.e., S_1 and S_2. These places are tasked to build the fixed order of the south and east phases.

$$M(G.S)+M(Y.S)+M(G.E)+M(Y.E)+M(S_1)+M(S_2)=1 \tag{5}$$

2.4 Control System of The Additional Green SignalsTime Interval

Control system of the additional green signalstimeinterval. A placeof a Petri net can be expanded into a subnet that contains many places andtransitions [18]. By expanding a place, a set of places can becreated and resulting in an expanded Petri net. Fig. 4.depicts the expansion ofa place G.S into places $G.S_1$, $G.S_2$, $G.S_3$, and $G.S_4$. This extension is useful for explaining the initial detaildesign of a high-level Petri net model [19].

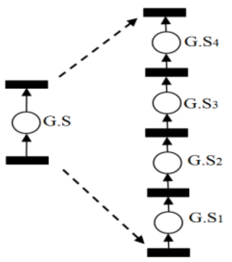

Fig. 4. Place G.S expansion become four places with each additional four-second time interval of $G.S_2$, $G.S_3$, and $G.S_4$, respectively [3].

$G.S_1$ represents the minimum green signal time interval while the traffic is low. All of the places $G.S_1$ $G.S_2$, $G.S_3$, and $G.S_4$ mean the maximum time interval of the green signal for the south arm traffic.

It is while the minimum green south signal time intervalis in the Invariant (6).The simulation is presented 24 seconds. It is illustrated as the top green in Fig. 5.

$$M(G.S_1)=M(G.S) \tag{6}$$

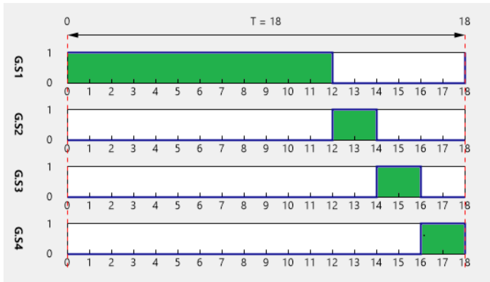

Fig. 5. The simulation result of the maximum green south signal time interval. Each step represented 2 seconds.

While one additional of four seconds time intervalis in the Invariant (7). It is presented as the green of the first and second row in Fig. 5.

$$M(G.S_1)+M(G.S_2)=M(G.S) \tag{7}$$

While two additional of four seconds time intervalis in the Invariant (8). It is the green at the first, second, and third row.

$$M(G.S_1)+M(G.S_2)+M(G.S_3)=M(G.S) \tag{8}$$

During the maximum green south signal time interval isin the Invariant (9). It is presented as all green.

$$M(G.S_1)+M(G.S_2)+M(G.S_3)+M(G.S_4)=M(G.S) \tag{9}$$

2.5 `Additional Green Signal Time Interval

The traffic signal implements the synchronous system on the railway doorstop. It illustrates the travel delays of vehicles coming from the southern arm which stopped by the railway doorstop closure. When the train is passing through, the traffic signal in the southern arm is turned red. The implication, the green signal on the east arm must be active. While the railway doorstop has been reopened, the signal on the southern arm implements additional green signal time intervals that canadapt to the traffic demand.

Additional green signal time interval. Each additional green signal time interval is four seconds [3]. This method in detail can be understood as the following expression. While the sensor does not detect a passing vehicle for three seconds, the controller instructsto end the green signal immediately and converts it to a yellow and then to the red signal. For one second remaining is the time for the process of deliveringthe messages from the sensor to the controller and the feedback response from the controller to the traffic signal. On the contrary, while in three seconds the sensor has recorded that at least onevehicle has passed, the controller commands for additional green signals.

While the vehicle traffic is very quiet, the queue of vehicles as a side effect of the barrier due to the closure of the railway doorstop is very small. This might happen like in the middle of the night. The control system commands the green traffic signal of the south arm tobe active at its minimum time interval only, even though this happens after the railway doorstop is reopened.

At the peak hours, the queue of vehicles after the railway passes is very large. This can happen during the day while the vehicle traffic volume is very high. The travel schedule in the southern arm is turned on using additional green signals at maximum time intervals.

When the vehicle queue ismoderate, the additional green signals of four seconds can be turned on once or twice. The green signal does not turn on at the minimum interval of time but also does not require to reach the maximum time interval.

2.6 Travel Delay

Travel Delay. AccordingtoKurniawan, F. and Al Hasibi, R.A. [20], traffic scheduling by the traffic signals or railway doorstop closure yields the waiting time for all vehicles arriving at the time t_d is $\omega(t_d)$.

$$\omega(t_d) = d(t_d) \times (t_o - t_d) \qquad (10)$$

Which $d(t_d)$ = traffic volume when the time t_d , t_o = the vehicles departure time, and t_d = the time when the vehicle arrives at the intersection.

Thetotal waiting time for all vehicles arriving while the red signals turn on is $\hat{\omega}(t_o)$.

$$\hat{\omega}(t_o) = \sum_{t=t_o-T}^{t_o-1} d(t) \times (t_o - t) \qquad (11)$$

Which $d(t)$ =traffic volume while the time t .

Table 2. Road Level of Service (LoS) index that is based on the delay of traffic flow

Level of Service (LoS) Index	Delay(Seconds/ PCU)
A	≤ 5,0
B	5,1 – 15,0
C	15,1 – 25,0
D	25,1 – 40,0
E	40,1 – 60,0
F	≥ 60,1

Cited from Tamin, O. Z. [21]

3. RESULT AND DISCUSSION

For measuring the volume of vehicles passing by on the road, its category refers to the Passenger Car Unit (PCU) standard metric [17], [21]. The categories are a motorcycle of 0.4 PCU, a low vehicle/ a passenger car of 1 PCU, and a heavy vehicle of 1.3 PCU, while unmotorized vehicles are ignored. From the result of the observation, the flow of vehicles in the east arm is 1.5 times the volume of traffic coming from the southern arm. The south arm capacity as the minor road is 1,200 PCUs/ hour and the east arm as the major road is 1,800 PCUs/ hour.

It is demonstrated that there is a railway doorstop closure for 150 seconds. This closure time interval is longer than a time interval of a traffic signal cycle. This happens in the middle of the night which the traffic is very low, in the dawn while the vehicle traffic ismoderate, and during peak hours at the day. The southern arm traffic variations apply a minimum green time interval as a fixed time strategy, an additional time interval of once, twice, and three times.

The signals on the eastern arm turn green when the train is passing through. The vehicles travel delay on the eastern arm is zero due to this method [8],[9].

Table 3. The vehicles queue on the south arm after the railway doorstop closure for 150 seconds

While The Time at	Traffic Volume (PCUs/ hour)	Green Time Interval (Seconds)	Queue of Vehicles (PCUs)	Vehicles Remaining (PCUs)	Average Delay (second/PCU)
Midnight	300	24	14	-	62
Dawn	450	24	21,5	-	72
Morning	600	32	29	-	79
Noon	900	36	44	8	86
Peak	1,200	36	59	23	99
Noon	900	24	44	20	103
Peak	1,200	24	59	29	126

The queue of vehicles increasesapproachinglinearly while its stop [16]. Based on Table 3, when the vehicle volume is 300 PCUs/ hour, there is a queue of 14 PCUs. This does not require additional green signals. The traffic crosses the intersection consecutively in a vehicle platoon after the railway doorstop is reopened. A fixed time strategy with a green signal of twenty-four seconds is too long. The average travel delay is 62 seconds / PCU.

At dawn, while the vehicle volume is 450 PCUs/ hour, there is a queue of 21.5 PCUs. The traffic signal does not require additional green signals.A twenty-four seconds time interval of the green signal has appropriated. The average travel delay is 72 seconds / PCU.

In the morning, while the vehicle volume is 600 PCUs/ hour, there is a queue of 29 PCUs. It requires twice additional green signals. The green signal time interval with an additional time is 32 seconds. The average travel delay is 79 seconds / PCU. Its score of LoS has not yet reached F.

When noon, the traffic volume is 900 PCUs/ hour. The vehicles queue after the railway doorstop closure is 44 PCUs.It requires the maximum time of the additional green signals, ie 36 seconds. The maximum of the green signal time interval is 36 seconds. The vehicles successfullycross the intersection of the first green signal is 36 PCUs. The vehicles remaining is 8 PCU and have to wait for the green signal in the next cycle. The average travel delay is 86 seconds/ PCU.

When peak hours, the traffic volume is 1200 PCUs/ hour. When the traffic flow is equal to the road capacity, it means that the vehicle's current has saturated. While the speed of the vehicles is inhibited, the efficiency decreases. In every cycle of the traffic signal always leave a lot of vehicles that have not passed while the green signal is on.

The railway creates a queue of 59 PCUs. This does not include vehicles that have not crossed while the green signal is lit in the previous cycle. It requiresthe maximum of additional green signals. The vehicles that have successfully crossed the intersection are 36 PCUs, and the remaining vehicles that have to wait for the green signal in the next cycle are 23 PCUs. The congestion occurs because the volume of vehicle traffic is very high. The average travel delay is 99 seconds/ PCU.

The queue of vehicles down 2 PCUs per traffic signal cycle with 72 seconds time interval. While the volume of vehicles passing on the road is not at peak hours, the queue of vehicles remaining each cycle continues to decline better.

Based on Table 3, when both noon and peak hours. While implementing a fixed time strategy that has a green signal for 24 seconds, the average delay is 103 seconds while the noon and 126 seconds when the peak hours. The average delayis higher than the traffic signal applying an additional green signal with a maximum time interval of 36 seconds.This means that the method with additional green signals is better.

Fig. 6. shows the result of the multiplication of the number of vehicles and the delay at the time interval of fifteen seconds. This is for 150 seconds when the railway doorstop closed while the train is passing through in the southern arm. The pattern is a transverse wave. It is related to the arrival of the vehicles that have a normal distribution. This means that the approximation of the average is the mode. Both arrival frequency of less than or more than the average is low. It uses a fixed time interval, i.e., three seconds.

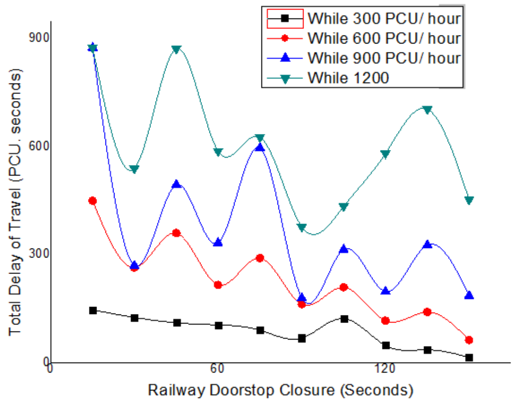

Fig. 6. The total delay of travel of vehicles of the south arm while the railway doorstop closure. Each step interval of data is 15 seconds.

In the beginning, all curves had a downward trend. The curves that have black and red lines are while the arrival volume 300 and 600 PCU/ hour, respectively. The vehicle arrival volume is less than or equal to half of the south arm's road capacity of 1200 PCU/ hour.

The blue and green lines are while the vehicles arrival volume 900 and 1200 PCU/ hour, respectively. The arrival volume is higher than half or equal to the road capacity of the southern arm. In ordinary circumstances, this means that there are remaining vehicles that have not yet crossed the intersection whilethe green signal turns on. In the last sixty seconds, the curve presented with the blue line shows no downward trend, but flat. This is similar to the curve given with the green line.It indicates a rising trenddduring peak hours. The reason, the vehicle that comes in the last sixty seconds have not crossed the intersection of the first green signal after the railway doorstop was reopened. They successfully crossed the intersection of the green signal in the second cycle. The curve indicates the travel delay is not reduced, it even increases.

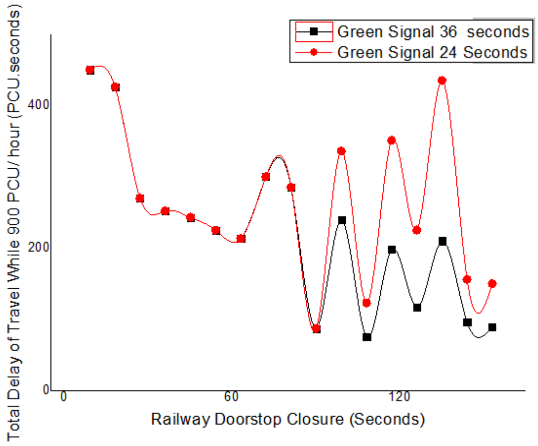

Fig. 7. The total delay of travel of vehicles of the south arm while implements green signal 36 and 24 seconds.

The volume of traffic is 900 PCU/ hour. The better Level of Performance is indicated by the lower of the travel delay. The performance of a traffic signal applying a green signal 36 is better than the signal that implements a 24-second interval time. Each step of data is nine seconds.

The additional green signal implementation can be performed on two or three traffic signal cycles after the closure of the railway doorstop. This solution is better to parse bottlenecks both when noon and peak hours. However, the additional green signals on the south arm may not cause new congestion in the eastern arm that has a larger traffic flow.

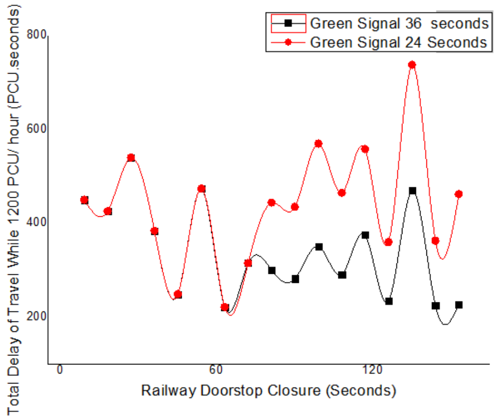

Fig. 8. The total delay of travel of vehicles of the south arm while implements green signal 36 and 24 seconds.

The volume of traffic is 1.200 PCUs/ hour. The performance of a traffic signal applying a green signal 36 is better than the signal that implements a 24-second interval time. Each step of data is nine seconds.

Table 4. The number of vehicles queues on the east arm while the green signal of the south arm after railway doorstop closure.

While The Time at	Traffic Volume (PCUs/ hour)	Green Time Interval (Seconds)	Queue of Vehicles(PCUs)	Vehicles Remaining (PCUs)	Delay Average (second/PCU)
Midnight	450	24	33	5	20
Dawn	675	24	33	7	26
Morning	900	32	41	12	27
Noon	1.350	36	45	16	30
Peak	1.800	36	45	21	31

Table 4 presents the average travel delay of vehicles coming from the east arm which increases due to the additional green signals in the southern arm. The red signal time interval in the eastern arm is the implication of the green signal in the southern arm. When the south arm signal is green or yellow, the red signal on the east arm must be active. It is aimed to separate the flow of vehicles coming from different directions to avoid conflict. Based on the comparison to the travel delays index in Table 2, the additional green signals when morning, noon, and peak doesnot decrease the index of road service levels, all index level are D.

Table 5. The average of the travel delayusing several variations of additional green signal time interval while peak hours

The arm	South Signal Green Time Interval(Seconds)				
	36	40	44	48	52
South (second/ PCU)	99	96	93	88	86
East (second/ PCU)	31	32	34	37	39

In Table 5, the maximum green signal time interval of 52 seconds. It can solve the congestion in the south arm while at noon and peak hours. The vehicle travel delay on the southern arm is turnsdown, but in the delay of vehicles coming from the east arm, its increases. The lowest score of the LoS for eastern arm performance is D.

While compared to Table 2, all vehicle travel delays coming from the southern arm after a train passing are greater than 60 seconds/ PCU or worse than category F of road level of service (LoS) index. The congestion during at noon of the day and peak hour cannot be solved at once cycle of the traffic signal. This requires several more cycles. This method is better when compared to the non-synchronous or ordinary synchronous traffic signal to the railway doorstop. The trains must be a top priority for the safety of travel.

4. CONCLUSION

The synchronous traffic signal to the railway doorstop applies a green signal to the east arm when the train passes in the southern arm. Theimplementationof the control system using Petri net of additional green signals for the queue of vehicles in the southern arm after the railway doorstop reopenedis an appropriate method to solve the congestion problem.Based on the analysis, the performance of the intersection become better because it can reduce travel delays.

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