DEVELOPMENT OF ABSTRACT ALGEBRA TEACHING MATERIALS BASED ON ABDUCTIVE-DEDUCTIVE STRATEGY

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ABSTRACT

This paper presents the results of research and development on abstract algebra teaching materials based on abductive-deductive strategy that applied in three steps. The first stepsare literature study, designing and developing teaching materials, expert validation, and revision. The second step is limited tests of eight students randomly selected. This test used the single-shot case study experimental design. The third step, product testing, conducted on a group of samples consisting of twentysix students. The method used in the third step is before-after experimental design. The instruments used are validation sheet, mathematical proof test, and performance effectiveness sheet. The data are analyzed using Wilcoxon Sign-Rank Test at 5% significance level and descriptive analysis. The results showed that the abstract algebra teaching materials based on the abductive-deductive strategy already meet the valid, effective, and practical criteria. It can be used in abstract algebracourses.

Keywords:*Teaching Materials, Abductive-Deductive Strategy, Abstract Algebra*

1. INTRODUCTION

Directorate of Education and Student Affairs (BELMAWA, 2013) describesthe learning outcomes for students of mathematics education based Indonesia National Qualifications Framework (KKNI) on the 6thlevel of qualification (on the second paragraph), consist of understanding either the general theoretical concept of knowledge or special one in depth, and able to formulate the procedural problem solving. Learning outcomes of this qualification is to master mathematical concepts to implement in primary and secondary education units as well as master the mathematical concepts for studying to the next level.

The learning outcomes that are intended in the curriculum are contained in analytical courses, such as the Abstract Algebra. This course does not study about calculations, but construction and validation the proof of a mathematical argument or conceptusing definitions and theorems in mathematics.Thus, by studying this course, students are able to think rationally, logically and systematically, and able to analyze the validity of an argument. In other words, through this course, students learn to develop their mathematical proof ability.

The ability of mathematical proving consists of ability to construct evidence and ability to validate the evidence (Selden & Selden, 2003). The ability to construct evidence includes the ability to use the methods of proof, definitions, lemma, and theorems to show the truth of a statement in mathematics (Abstract Algebra). While the ability to validate evidence includes the ability to critique evidence relating to the types of evidence that often appear in mathematics (Abstract Algebra). Thus, indicators measured in mathematical provingare: 1) using the method of proof, definitions, lemma,

and theoremsto show the truth of a statement in mathematics; 2) read a proof in mathematics to determine its error by looking at the compatibility among the axiom system, lemma, or theorem usingdeductive reasoning; 3) complete the proof correctly.

However, exploring students to master mathematical proving is not easy. Some researcher suggest that mathematical proving is a difficult ability for high school students (Maya & Sumarmo, 2011). The next, Moore (Maya & Sumarmo, 2011) categorizes the difficulties of mathematical proving ability into seven reasons, including: 1) students' ignorance of definitions; 2) students have only an intuitive understanding of mathematical concepts; 3) students' conceptual understanding is not enough to construct the proof; 4) the student is unable to generalize or give the example; 5) students do not know how to use definitions to obtain structured evidence; 6) students are unable to use the mathematical language or symbols; and 7) students do not know how to start proof.

The difficulty of developing mathematical proving ability can be seen in the abstract algebra course, especially in Mathematics Education Program at Muhammadiyah University of Sukabumi. It appears from the students' answers in the exam are still considered low-level thinking skills. Students are only able to remember some definitions without knowing how to construct evidence based on the definition that they already have. As a result, students often failure in constructing the proof.

Therefore, it is necessary an instructional innovation that is expected to make the students learn meaningful, so that the ability of mathematical proving can be trained and grown. One of the learning innovations can be realized through the development of teaching materials that specifically designed to explore students' mathematical provingin Abstract Algebra course.

The teaching materials developedare abstract algebra teaching materials based on abductive-deductive strategy, because some research results such as Kusnandi (2008) and Hiltrimartin & Hartono (Kusnandi, 2008) show that Abductive-Deductive strategy can develop the students' mathematical provingability in analytical courses, such as Number Theory and Abstract Algebra.

The Abductive-Deductive Strategy developed by Kusnandi (2008), is specifically designed to bring up the main idea of the evidentiary structure, both to understand existing evidence and to construct proof.

The framework of Adbuctive-Deductive strategy in theAction Process Object Schema (APOS) theorystarting from evidentiary problems can be simplified into a question"how to show the truth of the expected **final target"** based on a series of information provided in **the data.** The data and the final target are the two mental objects that are confronted to the students.

There are two actions that can be done directly when faced with the problem of proof. First,analyze the information provided in the **data,** and then compile them to produce **intermediate targets,** the next target is synthesized again to obtain the target of the next, and so on. These intermediate targets are another mental object that students may have had before. The process of obtaining the intermediate targets based on available data,called a **deductive process.**

The second action are to analyze the expected **finaltarget** and formulate an **intermediate target** from final target**,** so that based on a specific rule (definition or theorem) will arrive at the final target. The process of conditioning the intermediate targets to the the final target is a **process abductive key.**

Other process is to perform mental actions that can bridge between the results of deductive process (with intermediate targets)and abductive process results. That processes called **key processes.** The illustration of Abductive-Deductive framework in APOS theory, can be seen in the following figure.

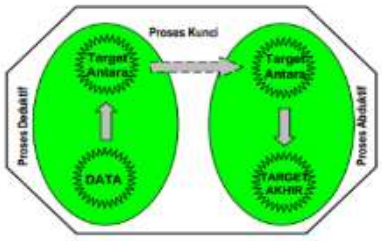

Figure 1. Abductive-Deductive Scheme

2. METHODS

This research was conducted from January until September2017 in Mathematics Education Program at Muhammadiyah University of Sukabumi. This research used Research and Development method. The procedure of this study is divided into three stages as follows.

a. The first stage consists of: 1) analyzing the abstract algebra teaching materials (including the course descriptions, learning outcomes, materials, and abductive-deductive strategy; 2) developing the product; 3) validating product using expert judgment with the Delphi technique; and 4) revision
b. The second stage is small group trial. At this stage, the abstract algebra teaching materials were used to eightrandomly selected Mathematics Education students as product users, with one shot case study design (Sugiyono, 2011). The researcher only taken one sample, then give a treatment by using abstract algebra teaching materials based on abductive-deductive strategy in the lesson, give the mathematical provingtest and performance effectiveness sheet in the end of the lesson.
c. The third stage is field trial. At this stage,abstract algebra teaching materialswere used by 26 students of abstract algebra course participants in Mathematics Education Program at Muhammadiyah University of Sukabumi. This field trial usedbefore-afterexperimental design (Sugiyono, 2011). The students given two treatments, learning without teaching materials (4 x 150 minutes) and learning by using abstract algebra teaching materials based on abductive-deductive strategy (4 x 150 minutes). Furthermore, at the end of the meeting, students are given mathematical provingtest and performance effectiveness sheet.

The research instruments used are validation sheet, mathematical proving test, and performance effectiveness sheet. Data were analyzedby descriptive analysis and Wilcoxon Signed Ranks Test at5% level significance.

3. RESULTS AND DISCUSSION

The abstract algebra teaching materials based on abductive-deductive strategy developed on four subject matter, group theory, subgroups,cosets and normal subgroup, and homomorphism of groups. Rate the validity of these teaching materials was carried out by expert judgment using

Delphi techniques.Assessment conducted by four lecturers of mathematics education continuously through a questionnaire to facilitate the group decision, without the need to fully meet face to face.

Expert validation of this abstract algebra teaching materials include fiveaspects: a) the feasibility of the material aspect consisting of twelve indicators; b) the feasibility of linguistic aspect consisting of four indicators; c) the feasibility of presentation aspect consisting of two indicators; d) the feasibility of teaching materials effect on student learning process consisting of three indicators; and e) the feasibility of the overall display aspect consisting of two indicators. So that, there are 23 indicatorsused in this assessment.Each indicator is assessed using 1-5 scale. Theassessment categories can be seen on the table 1 below.

Table 1. Teaching Materials Assessment Categories

Scale	Categories	Criteria
1	Bad	if the criteria are met less than 20%
2	Good Enough	if 20% -39% of criteria have been met
3	Good	scale if 40% - 59% criteria have been met
4	Very Good	if 60% -79% of criteria have been met
5	Excellent	80-100% of criteria have been met

The overall assessment of abstract algebra teaching materials based on abductive-deductive strategy obtained an average rating 4.35 of 5. it can be concluded that more than 80% criteria of abstract algebra teaching materials based on abductive-deductive strategy have been met and pertained the excellent category. So, abstract algebra teaching materials based on abductive-deductive strategyhas valid to use in learning. Recapitulation of this assessments based on the five aspects of teaching materials can be seen in the figure 2.

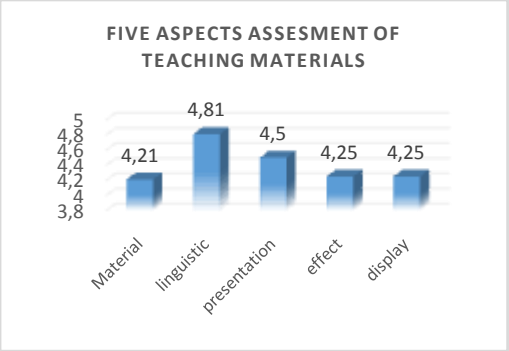

Figure 2. Teaching Materials Assessment RecapitulationBased on the Five Aspects

After having valid criteria, abstract algebra teaching materials based on abductive-deductive strategy is tested on a small group trial. The trial was conducted for two meetings (2 x 150 minutes).

The results of small group trial show that the achievement of students' mathematical proving ability increased 43.18%. This achievement can see in the following figure.

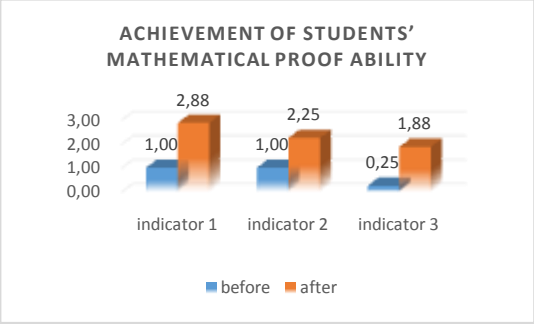

Figure 3. Achievement of Students' Mathematical Proving Ability Before and After Using Teaching Materials

Based on Figure 3, the students' mathematical proving ability based on each indicator has improved quite well. The first indicator (using the method of proof, definition, and theorem to show the truth of a statement in mathematics) has increased 62.67% and classified as a medium improvement category. The second indicator (reading a proof in mathematics to determine the truth or error by looking at the compatibility of the axiom system, the premise, or theorem) increased 71.43% and classified as high improvement category. And the third indicator, (complete the proof) has increased 59.27% and and classified as a medium improvement category. This result shows that the abstract algebra teaching materials based on abductive-deductive strategy are quite practical to be used in learning.

In addition, practical judgment is also performed on students' performance. The results of students' performance before and after using teaching materials can be seen in the following figure.

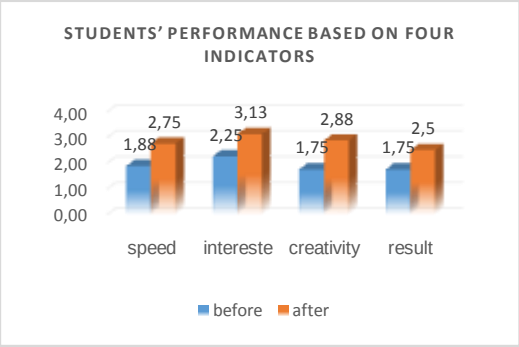

Figure 4. Students' Performance Before and After Using Teaching Materials Based on Four Indicators

Based on Figure 4, students' performance based on the speed of understanding, increased 41.04% and classified as medium improvement category. Students'performance based on students' interest in learning increased 50.28% and categorized as medium improvement. Students' performance based on students' creativity indicators after using teaching materials increased 50.22% and categorized as medium improvement. Students' performance based on learning result after using teaching materials increased 33,33% and classified as medium improvementcategory.

Based on these data, it can be concluded that the achievement of students' mathematical proof ability increased 43.18%. Thus, the abstract algebra teaching materials based on abductive-deductive strategyhave been classified as practical category, meaning there are valid and fit for use in the learning process in learning. However, even if it is worthy of use, the teaching materials are subsequently revised in the display aspect, adds sample illustrations, adds an example of proof, and adds exercise or structured tasks.

After meeting the valid and practical categories, then the teaching materials are tested again on a wider scale to see their effectiveness. The trial involved 26 students of abstract algebra courses participants and held for 8 meetings. The results of this field trial can be seen in Figure 5 as follows.

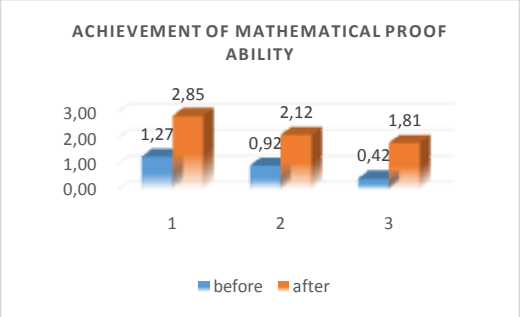

Figure 5. Achievement of Mathematical Proving Ability of Students Before and After Using Teaching Materials

Based on figure 5, the achievement of students' mathematical proving ability based on each indicator has increased. The achievement based on first indicator (using the method of proof, definition, and theorem to show the truth of a statement in mathematics) increased 57.88% and categorized as medium improvement. The achievement of second indicator (reading a proof in mathematics to determine the truth or error by looking at the suitability of the axiom system, premise, or theorem) increases 38.96% and categorized as a medium improvement. Achievement of indicator 3 (complete proof) increased by 53.88% and categorized as medium improvement category. So that, achievement of students' mathematical proof ability increased 37.76%.

To see a significant increasing statistically, then performed the Wilcoxon test at 5% significance level. The test results show that the significant value is 0.00. This value is less than 5% significance level, so it can be concluded that students' mathematical proof ability after using abstract algebra teaching materials based on abductive-deductive strategy is significantly better than before.

In addition, effectiveness assessment is also seen based on students' performance. The performance of students before and after using teaching materials based on four indicators also increased. This can be seen in Figure 6.

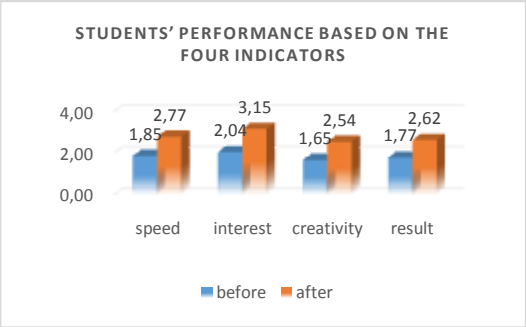

Figure 6. Students' Performance Before and After Using Teaching Materials Based on the Four Indicators

Based on Figure 6, it can be seen that the student's performance based on speed of understanding indicator after using teaching materials increased 42.79% and classified as medium improvement.Students' performance based on student interest in learning after using teaching materials increased 56.63% and categorized as medium improvement. Students' performance based on students' creativity after using teaching materials increased 37.87% and categorized as medium improvement. Students' performance based on learning outcomes obtained after using teaching materials increased 38.11% and classified as medium improvement. And overall student performance after using teaching materials increased by 23.56%and classified as medium improvement.

Based on the result, it can be concluded that teaching material has been classified as an effective category, especially in improving the ability of mathematical proof.

4. CONCLUSIONS

Based on the research result, the abstract algebra teaching materialsbased on the abductive-deductive strategy has valid, practical, and effective criteria. Valid criteria can be seen from the assessment of abstract algebra teaching material based on the abductive-deductive strategy. It has 4.35

from 5. So, more than 80% of this teaching materials suitable for learning. Practical criteria can be seen from the improvement of students' mathematical proof achievement that increased 43.18% and the overall assessment of student performance that increased 22.65% in a small group trial results. Effective criteria can be seen from the results of field trials. It shows that students' mathematical provingability after using abstract algebra teaching materials based on abductive-deductive strategy is significantly better than before. In addition, the effectiveness of student performance after using this teaching materials more increased 23.56% than before.

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