THE CHARACTERISTICS OF PROSPECTIVE TEACHERS' INTERPRETATION ABOUT STUDENTS MATHEMATICAL THINKING IN "COMPARING MODEL"

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Abstract

Students' mathematical thinking is an important thing to be known by prospective teachers. It required the ability to interpret the students' matematical thinking. Based on that, they can determine the appropriate learning decision-making. Comparing model is one of the interpretation models from Wilson, Lee, and Hollebrands. This article will describe the characteristics of prospective teacher interpretation about students' mathematical thinking on the model of interpretation. Subjects were selected by considering them in following the students' strategies in solving the problem.This study used qualitative methods to generate descriptive data interpretation characteristics of prospective teachers about students' mathematical thinking. Comparing model is a model of interpretation in which a person interprets student thinking based on student work. This study found 2 characteristics in comparing model thosewere comparing work and comparing knowledge. In the comparing work, the subject showed the characteristics of working on the problem first before providing interpretation and comparing the students' work with their own work. In the comparing knowledge, the subject showed the characteristics of not comparing the students' work with their work but with their knowledge or thought. Subjects did not solve problems in writing, but considered and explained how to solve in general.

Keywords:*interpretation, comparing model, comparing work, comparing knowledge*

1 INTRODUCTION

Teachers are people who have an important role in understanding students' thinking. Therefore, they need to interpret the students' mathematical thinking. "Interpretation ofstudents' mathematical thinking is giving impression, opinion, or a theoretical viewtowards mathematical information in the form of students' written work in solving problems"[5]. Their ability to interpret involves understanding their knowledge of the material itself. An understanding of how students work will lead them to an understanding of student thinking.

One of the most the fundamental goals of teaching mathematics is being able to use mathematical thinking in solving problems [3]. Nevertheless, it is also one of its most elusive goals. Mathematical thinking involves mental activity. It was facilitated by PSTs mathematics knowledge and their positive disposition towards mathematical problem solving. So, it is important to study about mathematical thinking.

Teachers have access to student learning activities both written and oral activities, but do not have direct access to students' mathematical thinking[1]. Teachers can only access evidence of their mathematical thinking such as student action or activity. Teachers can develop models of students' mathematical understandings by observing students' mathematical activities. That activities include hypotheses about what students know and understand [7]; [4]. Especially

for PSTS, their understanding of students' mathematical thinking will be determined much by what they understand or learn about textual theory or knowledge, little about the strategies that students undertake.

Teachers can interpret students' mathematical thinking by identifying strategies that students might use in problem solving. Teachers can also identify why certain problems become difficult and cause problems considering the characteristics of students' thinking.Prospective teachers (further we call PSTs) do not have much knowledge about the various strategies of students in solving the problem.They also have no access to the development of research results about the student's thoughts or strategies. Hence their understanding of student strategies is limited from their own theory or experience. Therefore, their interpretation of students' mathematical thinking will also depend largely on their knowledge and experience. They will compare students' actions with their own actions, either implicitly or explicitly[8]. This called comparing model analysis. Based on that, to interpret the mathematical thinking of the students, prospective teachers analyze by comparing, ie, equating or differentiating student strategy artifacts with their work or knowledge of the concept.This allows the emergence of two characteristics: comparing work and comparing knowledge.

2 THEORITICAL REVIEW

2.1 Mathematical Thinking

Currently the learning of mathematics deals a lot about how students "doing math" to gain an understanding of concepts. It'susually involves the application of procedures such as addition, subtraction, multiplication, division, estimation, and measurement to solve an algorithmic or story problem correctly and successfully. It's also involved symbolic manipulations. Doing math is all about the reproducing and applying facts and procedures to achieve the correct answer. Its different with mathematical thinking. Mathematical thinking, by contrast, is a specific way of thinking about things in the world [2]. Therefore mathematical thinking includes logical and analytic thinking as well asquantitative reasoning. They are a crucial but elusive ability.

In order to engage students doing math and thinking matemathically, teacher need to presented problemthat represent actual real world issues, problems, and situations. Its not just a story problem that has students do math in a real world context.

2.2 Comparing Model

Comparing is one of model-building process in interpreting students mathematical thinking. It's representing by two separatelyparallel boxes. They are a PSTs thinking (T_T) and their written work(T_W) in one box and a student's thinking(S_T) and their written work(S_W) in the other box[8].The arrows were use to represent a PST focusing on his or her attention. We used solid arrow to indicate explicit evidence of a PST's attention and dotted arrow to indicate an implicit of a PST's attention.

There are 2 types of PSTs's focus of attention, that is explicit and implicit attention. The explicitly attention is what PSTs attend to the student's written work or what students might think based on student work. Otherwise, implisitattention is inferred based on the PST's work. To modeled how PSTs analyze students' work,we read the diagram from the upper-left corner and following the arrows indicating their focus of attention.

The following is a picture of the process of constructing interpretations, comparing types from [8].

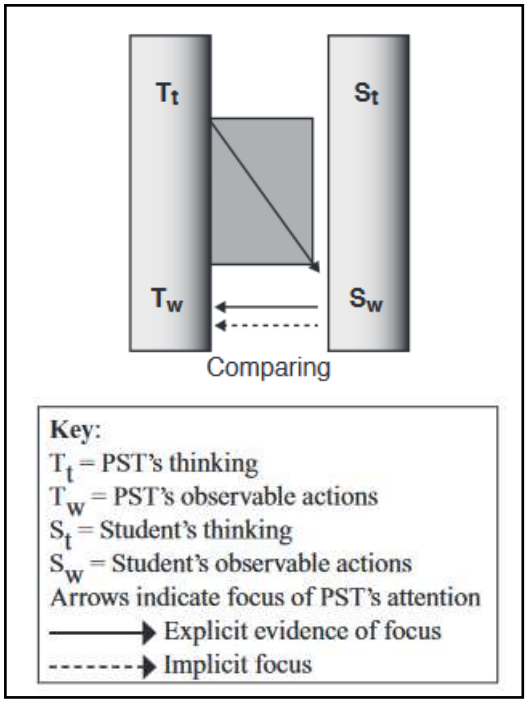

Figure 1: Comparing models to illustrate PSTs' analysis of students' work.

Comparing model is one of process that PSTs used to construct models of students' thinking[8]. In comparing model, PSTs compared students' activity with their own activity, either implicitly or explicitly. The model-building processes in this category include how the PST searched for similarities and differences between theirs and those that the PST noticed in the students' work.

Comparing between students' observable action and PSTs observable action can lead us to two different characteristic of interpretation. Students' action can be their written work or First, Comparing work is a characteristic when PSTs comparing students's action about the task with their own action. Their own action is their written work about the same task that student's done. The second is comparing knowledge, is characteristic when PSTs comparing students's action with his/her knowledge or teori about given problem. We can represent this ini following picture.

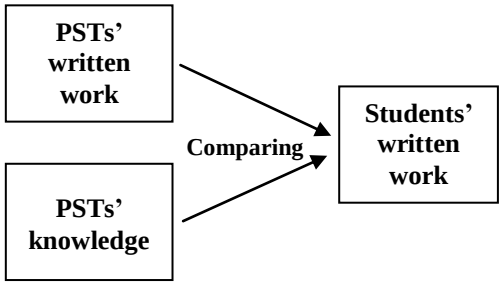

Figure 2: Type of Comparing

In this article, we tend to describe two characteristic of comparing as building process analysis to interpret student's mathematical thinking. By comparing artifact of a student's work onsolving problem PSTs could get understanding about students' mathematical thinking so they can interprete that.

3 METHOD

The study was conducted on third year students of Mathematics Education Program inMuhammadiyah University of Purworejo. The subject were pre-service teachers (PSTs) who can attend to at least 2 of 4 strategies that students do in solving Building Construction Problems(BCP). The student's strategies in completing BCPwere shown by the artifact of the student's written work.

We assign tasks to PSTs to attend to students' written work and explain what they understand about students' mathematical thinking based on student work. The task was written in Task of Interpretation of Students' Mathematical Thinking (ToIoSMT). Artifact of PSTs explanation of students' mathematical thinking technically we call PSTs interpretation.

We used third-order models from Wilson, Lee, and Hollebrands (2011) to describe our desciption of PSTs' interpretation of students' mathematical thinking (see Figure 3).

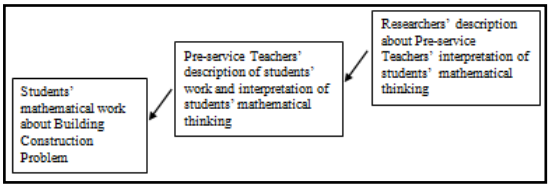

Figure 3: A representation of the Process of Researchers Characterising PSTsinterpretation of students' mathematical thinking

This is similar to the point of view of [6] in which they explain "the teacher's perspective from the researchers' perspectives" (p. 254).In this model, we describe the characteristik of PSTsinterpretation of students' mathematical thinking based on their written work and think aloud. The researchers'descriptionabout characteristicswas strengthened by the interviews.

4 RESULTS AND DISCUSSION

Researchers provide 4 kinds of written work students in completing the BCP. Here is a BCP that students have done.

> In order to construct a building, the contractor takes 15 months with 120 workers. For a reason, the contractor wants a 3 month accelerated job. If the ability to work for each worker is the same and that the project can be completed on time, how many workers should beadd.

We provided 4 samples of students' written work in completing BCP.Student A and D completed BCP using inverse ratio. Student A completedBCP by writing down the method used that was the ratio of reversed value as well as the linkage of information time and many workers. Student A did not write down the steps to get the number 150. The student simply wrote that many of the added workers were 150 - 120 = 30 workers.Whereas, student D writed in detail the calculation of inverse ratio. Students B and C both completed the BCP by using direct proportion. Student B divided 12 by 15 and multiplies the result by 120. He multiplied the time ratiobythe number of workers.While student C writes in detail the steps of BCP problem solving: known information, completion plans, direct proportion calculations, and conclusions.

4.1 Comparing Work

The subject begancompleting ToIoSMT by reading the BCP and attending to student's strategies. Ar and Sal have shown evidence that they use comparing work to interpret students' mathematical thinking. Both complete the BCP to be compared with the students' written work artifacts.Ar completed the BCP first before attending to the student's work, while Sal completed the BCP after following the student's work. Although Sal solves BCP problems after attending to student work, both used their work explisitly to determine what was true among the four students ie. A and D.

Ar usedhis work to determine which of the four student jobs is correct. According to him, the number of workers to be added is 30 people and there are two correct answers. This is evident from the following aloud quotes.

> If according to my work, e... additional workers will be 30 people. But here are different. Two students answered correctly.

He said "if according to my work", it shows that he compared the student's work with his own work.At the time of interpreting the mathematical thoughts of student A, Ar showed evidence that by completing the BCP he couldpresume students' unwritten thoughts.Ar's interpretation of student A's mathematical thinking is that student A understands what is known and what to look for even though the student does not perform a detailed calculation to obtain 150. Although the calculation process for obtaining 150 was not written, Ar implicitly assumes that students use inverse ratio. It was apparent from what Ar submitted in the interview passage.

> ... If I model the A student's work, A student already knows that it uses the concept of inverse ratio. .... Na, to get 150, he did not write. It's quite confusing, where he gets 150. That possibility is 150 from ... can ... it's reversed. It can be reversed or 15 multiplied by 10. Possible as it is.

This is reinforced by the explanation when asked where to get the number 10.

> ... Students guess, there is a number 12, there is a number 120, there is a number 15 (refers to student A's work on the part of the relationship between time and many workers). Possibility 15 multiplied by 10.

In the beginning, Sal attend to the students' written work according to what the students wrote without giving an argument. Next Sal used gestures while counting. It looks from the following Sal's behavior and think aloud.

> Student's thinking ... it should be ... (scrutinize and mumble, move hands doing calculations) then 15 per 12 equals x per 120.

Sal by herself also completed the BCP at the timeof interpreting students' mathematical thinking. This is shown in the following interview passage.

> R : Later on, you do these doodles. When did you do it?
>
> Sal : Here I am ... at the time of doing the second task. I see the result is true or not. So I doodle myself.

Sal used her work to convince herself about the details of student's strategies and mathematical thinking in completing the BCP. Sal saw that the work or the completion of each students is different. Further, Sal used her work to see which students are already understand and which ones are not. This is evident from the following interview excerpt.

> Me : Further you assure which one is right which one is wrong with?
>
> Sal : with ... writing down the doodles.
>
> R : Means you look back to .. (Sal reply: student work) heeh... then doodled,then?
>
> Sal : to conclude which one ... students who already understand, which ones have not.

From the two subject, characteristic of interpretation by comparing work were:

a. completed the BCP both at the beginning and end of the interpretation process;
b. checked out the right or wrong work of the student based on his/her work;
c. recognized student strategies that match their strategy.

Their interpretationon students' mathematical thinking characterized by a description of what students understand and what they do not understand. This is based on the set of student strategies they follow and describe the process.

4.1 Comparing Knowledge

Hap dan Hen have shown evidence that they were comparing student work with their knowledge in analyzing student work during the aloud process. They did not complete the BCP by doodles to be compared with the students' written work artifacts but compared it with their knowledge.

Hap showed evidence that he was doing analysis with comparing knowledge. She compared the student's work with his knowledge to interpret the students' mathematical thinking in completing the BCP. Hap began by reading the questions and proceed with the students' work. When looking at student A's work, Hap first thinks about the solution. In this case Hap only usedherknowledge, not by solving the PG problem in writing. At student A's work, student A did not write an operation that shows aninverseratio. When looking at student A's work, Hap considered her knowledge of inverse ratio, ie comparison

between variables, ie the time ratio is equal to the number of people. By multiplication will be the number of people needed. While the number of workers added is obtained by reducing the number of workers should be with many existing workers. This is apparent when Hap looks at the work of Student A on the following transcript think aloud.

> .... Using the concept Ta: Tb is the same as person a: person b. Later will result a numberof person b equal to a number of person a multiplied by Tb and divided by Ta. So the number of workers to be added is by reducing the person who met the person who is in the second time. The additional person is the second person minus the first person. That will generate the amount.

Hap knew the right answer not by doing in writing but rather using his thinking (knowledge). She did it to match students' work with their thinking. In this way Hap becomes aware that the correct answer belongs to student A and student D.This is evident from the following interviewexcerpt.

> R : Are you scratching or not?
> Hap: No.
> R : why?
> Hap: Because, from the student's work I'm direct ... from this work I was told to give a comment, I look at it from this answer. And I match with my thinking. Yes this is with ... such fondness ... and I get the correct answer between this one (A) and this (D). The point is these onesare true.

Hen showed evidence that he was doing analysis with comparing knowledge. When looked at the students' work, Hen did not know the students' answer right or wrong. He analyzed by considering the formula that should be used. It wasapparent when Hen looks at the student's A work on the following think aloud.

> The ratio used ... In the ratio formula there are two direct ratio and inverse ratio. The proportion used (while looking at student work) on the problems presented is a inverse ratio.

In the interview, Hen explained herknowledge of the theory of inverse ratio. In inverse ratio, the formula used is A_1: B_2 equal to A_2: B_1.

> Since it is presented that the ratio is reversed then the student should use a formula ... for example, using the ratio A1 : B2 equals A2 : B1 so. Na, this writing is appropriate.

From the two subject, characteristic of interpretation by comparing work were:

a. did not completed the BCP by wrote in paper;
b. checked out the right or wrong are the student work by their own knowledge based on student work;
c. recognized student strategies that match their strategywith their understanding .

Their interpretation on students' mathematical thinking characterized by a description of what students understand and what they do not understand. This is based on the set of student strategies they follow and their own knowledge on that problem.

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5 CONCLUSION

Based on data analysis, our findings are that the category of Comparing Model analysis can be distinguished by 2 comparing models, namely comparing work and comparing knowledge. The two analytical models compare the actions of the PsTs with the student's actions. What distinguishes between the two is the use of doodles or written work and unwritten. In the comparing work, PSTs completed BCP in written, while the PSTs did not. They used their thinking or knowledge to analyze or interpret but do not express it

in writing. The characteristic of their interpretation of students' mathematical thinking is a description of what students understand and what they do not understand based on the set of student strategies they follow and the process of completing the BCP.

REFERENCES

[1] E. von Glasersfeld, 1995, A constructivist approach to teaching. In: Steffe L. P. & Gale J. (eds.) Constructivism in education, Erlbaum, Hillsdale: 3–15.Available at http://www.vonglasersfeld.com/172

[2] K. Devlin,2012,Introduction to MathematicalThinking, Retrieved from: www.mat.ufrgs.br/~portosil/curso-Devlin.pdf

[3] K. Stacey, 2006, What Is Mathematical Thinking and Why Is It Important?Available at: http://www.criced.tsukuba.ac.jp/math/apec/apec2007/paper_pdf/Kaye%20Stacey.pdf

[4] L.P. Steffe, & P.Thomson, 2000,Teaching experiments methodology: Underlying principles and essential characteristics, in E. Kelly & R. Lesh (eds) *Research Design in Mathematics and Science Education*, Hillsdale, Nj: Laurence Erlbaum

[5] M. Sapti, Purwanto, M. Sri, and E.B. Irawan, 2016, Proceeding of 3rdInternational Conference on Research, Implementation and Education of Mathematics and Science Yogyakarta, Pre-service Teacher Interpretations ofStudents' MathematicalUnderstanding.

[6] M. Simon, &R. Tzur, 1999, Explicating the teacher's perspective from the researchers' perspectives:generating accounts of mathematics teachers' practice. *Journal for Research in MathematicsEducation*,30(3), 252-264.

[7] P.Cobb & L. P. Steffe, 1983, The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education* 14(2): 83–94. Available at http://cepa.info/2096

[8] P.H. Wilson, H.S. Lee,, & K. F. Hollebrands, 2011, Understanding Prospective Mathematics Teachers' Processess for Making Sense of Students' Work with Technology. Jurnal for Research in Mathematics Education. Vol.42 Number 1 page 39-64. Retrieved from 120.164.46.147 at March 16, 2016. 22.07.20 UTC